

String Junction Model, Cluster Hypothesis, Penta-Quark Baryon and Tetra-Quark Meson

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Thirty years ago we proposed string junction model of hadrons and examined structure and reaction of hadrons including exotic ones. Mass m of exotic hadrons of light quarks is roughly given by $m \sim N_J \cdot m_B$, where N_J is the total number of junctions and $m_B \sim 1$ GeV is the ordinary light baryon mass. In this paper we introduce “cluster hypothesis” into the model by which mass of a complex hadron is given by the sum of masses of clusters composing it. The hypothesis guarantees the established picture that mass differences of hadrons of the same string junction structure are due to those of the constituent quarks.

A candidate for penta-quark baryon $\Theta(1530 \text{ MeV}, S = +1)$ including a strange anti-quark \bar{s} and that for tetra-quark meson $Z^+(4430 \text{ MeV})$ recently reported by the Bell collaboration are examined in parallel. Θ is considered to have non-strange partners, which are lighter by the mass difference Δ_s between strange and non-strange quarks. Mass of such light penta-quark baryons with $N_J = 3$ is expected to be about 3 GeV. Several parameters of the model are estimated such as mass of junction of $m_J \sim O(10) \text{ MeV}$. While mass of light tetra-quark meson with $N_J = 2$ is expected to be about 2 GeV, $Z^+(4430 \text{ MeV})$ containing (u, c, \bar{d}, \bar{c}) gives a clue to determine some parameters of the model, e.g., inter-junction string energy m_{IJ} .

§1. Introduction

There exist numbers of ordinary hadrons made from quark q and anti-quark \bar{q} : the ordinary meson $M = q\bar{q}$ and the ordinary baryon $B = qqq$. A discovery of unconventional hadrons with extraordinary structure, however, had not been reported for a long time. This poses a quite keen contrast with situations of atoms and atomic nuclei both as physical composite systems. The reason of the contrast may be closely related to the mechanism of quark confinement. Since we have almost no established methods to treat low energy phenomena of QCD, we adopt in this paper a model based on the string picture of confinement. We expect this will give a clue to clarify low energy nature of QCD in a complementary way to other approaches such as lattice gauge theory.

In our model quark confinement is considered to be realized by colored string. The string has “orientation” because, when the string is cut by pair creation of quarks, the sequence of q and \bar{q} is unique. When we define the orientation by the direction toward a confined quark at the end, there should exist in the baryon a

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“singular point” from which three strings emerge and where three colors are neutralized. This point is called “junction”.¹⁾ In the string junction model we proposed in 1977²⁾ (abbreviated as SJM)(see also 3),4),5)), we investigated unconventional hadrons including exotic ones. The reason why they were so difficult to be observed was attributed to their complex structure, in particular, to the nature of junction, which was regarded as a physical entity similarly to quarks.

In 2003 a discovery of an exotic penta-quark baryon, $\Theta(1530)$ with strangeness $S = +1$, was reported,⁶⁾ though experimental judge for this candidate is controversial up to now. Recently the Belle Collaboration has reported a new resonance $Z^+(4430)$.⁷⁾ As emphasized in the paper, Z^+ is a charged particle and is a good candidate for tetra-quark meson containing (u, c, \bar{d}, \bar{c}) . In the present article, we investigate exotic hadrons of SJM.

Our paper is organized as follows. In section 2, a brief summary of hadrons of SJM is firstly given. Except for the ordinary meson, baryon and a simple string ring, every unconventional hadron has a chain of junctions and anti-junctions as its skeleton. Under reasonable assumptions the standard mass m of a hadron containing only light quarks is roughly given by

$$m \sim N_J \cdot m_B, \quad (1.1)$$

where N_J is the total number of junctions of the hadron and $m_B \sim 1$ GeV is the ordinary light baryon mass. Then the mass of light penta-quark baryon with $N_J = 3$ is supposed to be about 3 GeV while that of light tetra-quark meson with $N_J = 2$ to be about 2 GeV.

We introduce “cluster hypothesis” into SJM. Namely, when all the inter-junction strings within a hadron are cut at the same time, each connected part is called as a single cluster. By the hypothesis mass of a hadron with multi-cluster structure is given by the sum of masses of the clusters, and the traditional picture that mass differences of hadrons with the same string junction structure come from those among constituent quarks is guaranteed. For ordinary mesons and baryons of light quarks, the mass difference between strange and non-strange quarks is expressed by the mass formula of Gell-Mann-Okubo(GMO)⁸⁾⁹⁾ with T_3^3 breaking of flavor $SU(3)_f$. We consider T_3^3 breaking should be applied to each cluster but not to any unconventional hadrons with complex structure as a whole. A comparison of exotic hadrons of SJM with those of Skyrmin model¹⁰⁾¹¹⁾ and di-quark model¹²⁾ is also given.

In section 3 we discuss quark rearrangement diagram, duality and selection rules, where physical roles of junction are shown in connection with tetra-quark meson and penta-quark baryon. Although “indirect”, \bar{B} - B duality gives important information that the mass of light tetra-quark meson should be about $2m_B$. The selection rule by Freund-Waltz-Rosner¹³⁾ to suppress the decay of tetra-quark meson into two ordinary mesons is replaced by a rule to forbid junction hair pin line diagram,¹⁴⁾ which is in parallel to the OZI rule¹⁵⁾¹⁶⁾¹⁷⁾ to forbid quark hair pin line diagram. This would explain the “narrow” width of Z^+ .

In section 4, the mass of junction is estimated as $m_J \sim O(10)$ MeV. A heuristic argument by uncertainty relation is given on the mass of string attached to a source quark together with its length in the ground state hadron. When $Z^+(4430)$

is established, it helps to determine basic parameters of SJM in a “direct” way. In subsection 4.4 newly added after the report of Z^+ , we identify it with tetra-quark meson composed from $(u, c, J; \bar{J}, \bar{d}, \bar{c})$ including a pair of junctions, and estimate the mass m_{IJ} of inter-junction string and the energy δ necessary to cut it by creating a pair of quarks. This makes it possible to predict masses of unconventional hadrons, such as 3380 MeV for Θ containing an \bar{s} . A summary of the parameters of SJM is presented in section 5.

§2. String junction model, cluster hypothesis and mass of hadrons

2.1. String junction model

In SJM there are various hadrons such as shown in Fig. 1. In this paper, the upper suffix of hadron symbols denotes N_J , the total number of junctions (sum of junction number and anti-junction number), and the lower suffix denotes N_q , the total number of quarks (sum of quark number and anti-quark number).

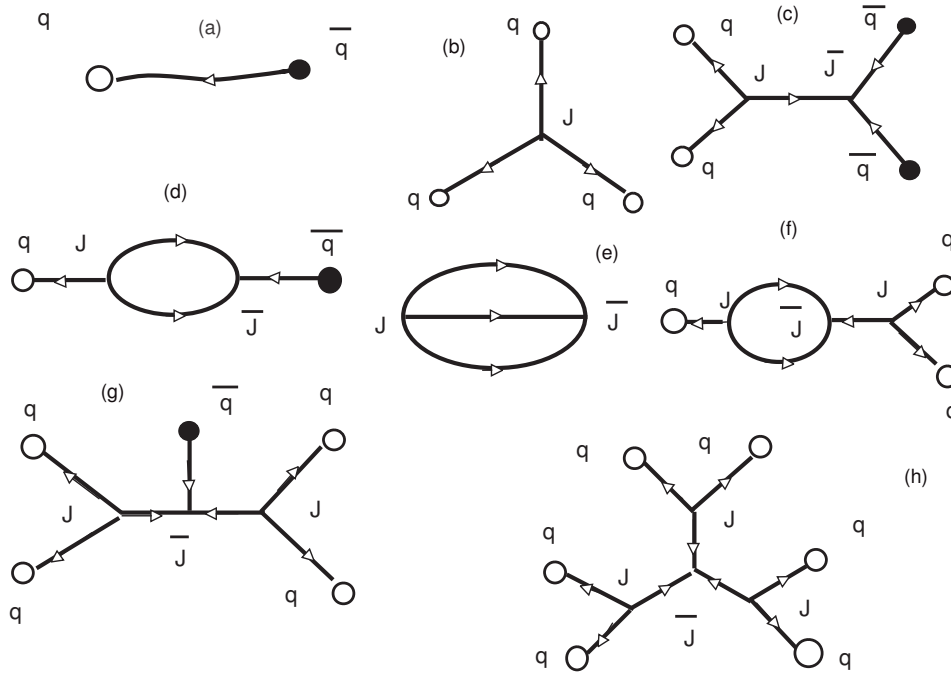


Fig. 1. Hadrons in string junction model. (a)ordinary meson M_2^0 , (b)ordinary baryon B_3^1 , (c)exotic meson M_4^2 , (d) M_2^2 , (e)gluonium S_0^2 , (f) B_3^3 , (g)exotic baryon B_5^3 , (h)di-baryon D_6^4 . Upper suffix and lower suffix denote the total numbers of junctions and quarks respectively.

1. Meson octet $M(8)$ and nonet $M(9) = M(8) \oplus M(1)$ have the structure $M \equiv M_2^0$ in which q and \bar{q} are connected with each other by an oriented string (Fig. 1(a)). Baryon octet $B(8)$ and decuplet $B(10)$ have the structure $B \equiv B_3^1$, in which three quarks are connected with a junction J (Fig. 1(b)).
2. A meson whose quantum number is realized only by four quark system $qq\bar{q}\bar{q}$

is expressed as M_4^2 in SJM. This exotic meson contains a pair of junctions (J and \bar{J}), and qq and $\bar{q}\bar{q}$ are connected with J and \bar{J} respectively. Junction and anti-junction are connected by an “inter-junction string”(Fig.1(c)). Exotic baryon B_5^3 contains $qq\bar{q}qq$ and $J\bar{J}J$, and the latter are connected by two inter-junction strings. Quarks qq and anti-quark \bar{q} are connected with J and \bar{J} respectively(Fig. 1(g)).

3. A string ring which contains neither quark nor junction is called “Pomeron”. The exchange of this in t -channel leads to diffraction scattering at high energies.
4. Except for ordinary mesons, baryons and Pomeron, hadrons in SJM contain at least one inter-junction string. It should be noted that unconventional hadrons have such skeleton type structure of multi-junction. M_2^2 (Fig. 1(d)), S_0^2 (Fig.1(e)) and B_3^3 (Fig.1(f)) are examples of unconventional hadrons.
5. In SJM, di-baryon D_6^4 is not merely six quark state but it has the structure with three junctions and one anti-junction(Fig1(h)).

2.2. Cluster hypothesis and mass of hadrons

Now we will introduce “cluster hypothesis” into SJM as (I) to (IV) below:

- (I) The ordinary meson M and baryon B , which do not contain inter-junction string, are defined as a single cluster.
- (II) **When all the inter-junction strings within a hadron are cut at the same time, each connected part is defined as “a single cluster”. Let N_{IJ} be the number of inter-junction string connecting J and \bar{J} . Any hadron with $N_{IJ} \neq 0$ has multi-cluster structure.**
- (III) It may be natural in SJM to express mass of hadron as¹⁸⁾¹⁹⁾²⁰⁾

$$m = m_q N_q + m_J N_J + m_{IJ} N_{IJ}, \quad (2.1)$$

where m_q denotes the “constituent quark mass”, which includes both the mass \tilde{m}_q of the “source quark” and the energy of the string connecting it with a junction. (A relation between m_q and \tilde{m}_q is discussed in subsection 4.2.) When we assign $N_{IJ} = 1/2$ to each of the “cut” inter-junction string, mass of any multi-cluster hadron is given by the sum of mass of the clusters (“additivity”):

$$m \sim \sum_{\text{cluster}} m_{\text{cluster}}. \quad (2.2)$$

- (IV) Since the mass of junction and the mass of inter-junction string are flavor independent, the mass of multi-cluster hadron faithfully reflects the constituent quark mass.

Except for the ordinary meson, baryon and Pomeron, there are three kinds of clusters: $\{qqJ \cdot (\text{HIJ})\}$, $\{qJ \cdot 2(\text{HIJ})\}$ and $\{J \cdot 3(\text{HIJ})\}$ together with their charge conjugate partners, where HIJ is the abbreviation of one half of the inter-junction string. Mass of them is shown in Table I. Several examples of cluster structure including exotic hadrons are given in the first column of Table II. For examples, exotic meson

Table I. Mass of clusters

cluster	N_q	N_J	N_{IJ}	m_{cluster}
$M_2^0(\bar{q}q)$	2	0	0	$2m_q$
$B_3^1(qqJq)$	3	1	0	$3m_q + m_J$
$qqJ \cdot (\text{HIJ})$	2	1	1/2	$2m_q + m_J + 1/2 \cdot m_{IJ}$
$qJ \cdot 2(\text{HIJ})$	1	1	$1/2 \times 2$	$m_q + m_J + m_{IJ}$
$J \cdot 3(\text{HIJ})$	0	1	$1/2 \times 3$	$m_J + 3/2 \cdot m_{IJ}$

M_4^2 is composed of two clusters, $\{qqJ \cdot (\text{HIJ})\}$ and $\{\bar{q}\bar{q}\bar{J} \cdot (\text{HIJ})\}$, and S_0^2 represents the gluonium without quark and is composed of two clusters $\{J \cdot 3(\text{HIJ})\}$, $\{\bar{J} \cdot 3(\text{HIJ})\}$. Penta-quark baryon B_5^3 is composed of three clusters, $\{qqJ \cdot (\text{HIJ})\}$, $\{\bar{q}\bar{J} \cdot 2(\text{HIJ})\}$ and $\{qqJ \cdot (\text{HIJ})\}$. Likewise, B_3^3 is composed of three clusters, $\{qqJ \cdot (\text{HIJ})\}$, $\{\bar{J} \cdot 3(\text{HIJ})\}$ and $\{qJ \cdot 2(\text{HIJ})\}$.

The mass of hadrons of Eq. (2.1) is rewritten as

$$m = m_B \cdot N_J - \delta \cdot N_{IJ} \quad (2.3)$$

if we note the relation

$$N_q = 3N_J - 2N_{IJ}, \quad (2.4)$$

where

$$m_B = 3m_q + m_J \quad (2.5)$$

denotes the mass of the ordinary baryon B and

$$\delta = 2m_q - m_{IJ} \quad (2.6)$$

means that the energy necessary to cut an inter-junction string by quark pair creation is δ .

The cluster structure of unconventional hadrons in Fig. 1 and standard mass of them are shown in Table II.

It may be natural to assume parameters m_q, m_J, δ, m_{IJ} are “universal” in SJM irrespective of hadron structure. If the parameter δ is small compared with m_B , multi-cluster hadron mass is approximately given by

$$m \sim N_J \cdot m_B. \quad (2.7)$$

Actually, we will see in section 3 and subsections 4.3 and 4.4 that the parameter δ is rather small. Then mass of tetra-quark meson and mass of penta-quark baryon are $\sim 2m_B$ and $\sim 3m_B$, respectively.

Table II. Standard mass of unconventional hadrons

Unconv. Meson	B	N_q	N_J	N_{IJ}	mass	cluster structure
$M_4^2(qqJ; \bar{J}\bar{q}\bar{q})$	0	4	2	1	$2m_B - \delta$	2 clusters
$M_2^2(qJ; \bar{J}\bar{q})$	0	2	2	2	$2m_B - 2\delta$	2 clusters
$S_0^2(J; \bar{J})$	0	0	2	3	$2m_B - 3\delta$	2 clusters
Unconv. Baryon	B	N_q	N_J	N_{IJ}	mass	
$B_5^3(qqJ; \bar{q}\bar{J}; Jqq)$	1	5	3	2	$3m_B - 2\delta$	3 clusters
$B_3^3(qqJ; \bar{J}; Jq)$	1	3	3	3	$3m_B - 3\delta$	3 clusters
Di-baryon	B	N_q	N_J	N_{IJ}	mass	
$D_6^4(qqJ; \bar{J}; Jqq; Jqq)$	2	6	4	3	$4m_B - 3\delta$	4 clusters

2.3. Mass splitting due to strange quark

Mass splitting due to strangeness among hadrons belonging to an irreducible $SU(3)_f$ representation is given by T_3^3 breaking of the GMO mass formula as

$$m = a + bY + c\{I(I+1) - \frac{1}{4}Y^2\}, \quad (2.8)$$

where I denotes isospin and $Y \equiv B + S$ does hypercharge, B and S being baryon number and strangeness respectively. The physical content of the T_3^3 breaking is clearly understood by the famous equal mass splitting of decuplet baryon $B(10)$. For decuplet representation $I = Y/2 + 1$, so Eq.(2.8) is a linear function of Y or $S = Y - 1$ and mass splitting of hadron is reduced to that of the constituent quarks:

$$\Delta_s = m_s - m_u \sim m_s - m_d. \quad (2.9)$$

For the cluster $\{qqJ \cdot (\text{HIJ})\}$, there are two cases for the quark pair qq . For the anti-symmetric case $[q, q]$ which belongs to $\bar{\mathbf{3}}$ representation of $SU(3)_f$, $I = -Y/2 + 1/3$ while for the symmetric case $\{q, q\}$ which belongs to $\mathbf{6}$ representation, $I = +Y/2 + 2/3$. Again Eq.(2.8) is linear on S for either case. For completeness, note that the cluster $\{qJ \cdot 2(\text{HIJ})\}$ of course belongs to $\mathbf{3}$ representation, thus has linear dependence on S . In this way, the mass splitting of hadrons in SJM under the cluster hypothesis is always reduced to the number of s and \bar{s} quarks contained.

The configuration of penta-quark baryon B_3^5 is

$$|(qq)J; \bar{J}\bar{q}; (qq)J\rangle \quad (2.10)$$

where (q, q) denotes either $[q, q]$ or $\{q, q\}$. For simplicity we take the same mass of

$2m_q$ for the both cases. If we write concretely, we have

$$\left\{ \begin{array}{ll} |(\mathcal{NN})J; \overline{\mathcal{N}}\bar{J}; (\mathcal{NN})J\rangle & (S=0), \quad m=3m_B-2\delta \\ |(s\mathcal{N})J; \overline{\mathcal{N}}\bar{J}; (\mathcal{NN})J\rangle & (S=-1), \quad m=3m_B-2\delta+\Delta_s \\ |(ss)J; \overline{\mathcal{N}}\bar{J}; (\mathcal{NN})J\rangle & (S=-2), \quad m=3m_B-2\delta+2\Delta_s \\ |(s\mathcal{N})J; \overline{\mathcal{N}}\bar{J}; (s\mathcal{N})J\rangle & (S=-2), \quad m=3m_B-2\delta+2\Delta_s \\ |(ss)J; \overline{\mathcal{N}}\bar{J}; (s\mathcal{N})J\rangle & (S=-3), \quad m=3m_B-2\delta+3\Delta_s \\ |(ss)J; \overline{\mathcal{N}}\bar{J}; (ss)J\rangle & (S=-4), \quad m=3m_B-2\delta+4\Delta_s \\ |(\mathcal{NN})J; \overline{s}\bar{J}; (\mathcal{NN})J\rangle & (S=+1), \quad m=3m_B-2\delta+\Delta_s \\ |(s\mathcal{N})J; \overline{s}\bar{J}; (\mathcal{NN})J\rangle & (S=0), \quad m=3m_B-2\delta+2\Delta_s \\ |(ss)J; \overline{s}\bar{J}; (\mathcal{NN})J\rangle & (S=-1), \quad m=3m_B-2\delta+3\Delta_s \\ |(s\mathcal{N})J; \overline{s}\bar{J}; (s\mathcal{N})J\rangle & (S=-1), \quad m=3m_B-2\delta+3\Delta_s \\ |(ss)J; \overline{s}\bar{J}; (s\mathcal{N})J\rangle & (S=-2), \quad m=3m_B-2\delta+4\Delta_s \\ |(ss)J; \overline{s}\bar{J}; (ss)J\rangle & (S=-3), \quad m=3m_B-2\delta+5\Delta_s. \end{array} \right. \quad (2.11)$$

For the pair of same flavor quarks, uu, dd and ss , only the symmetric combination $\{q, q\}$ survives.

Characteristic features of penta-quark baryon of SJM are as follows:

- (1) If we neglect both Δ_s and δ , which may be expected to be small compared with m_B , exotic baryon B_5^3 will have mass around three times of baryon mass:

$$m_{B_5^3} \sim 3m_B \sim 3 \text{ GeV}. \quad (2.12)$$

- (2) If we take into account the mass breaking $\Delta_s = m_s - m_u \sim m_s - m_d = 130 \sim 150 \text{ MeV}^*)$, we have

$$m(|(\mathcal{NN})J; \overline{\mathcal{N}}\bar{J}; (\mathcal{NN})J\rangle) < m(\Theta = |(\mathcal{NN})J; \overline{s}\bar{J}; (\mathcal{NN})J\rangle), \quad (2.13)$$

with the mass difference Δ_s .

- (3) In SJM, Θ can be neither the lightest of penta-quark baryons nor be so light as 1530 MeV.

Here we will add short comments on some other models of penta-quark baryon; Skyrmon model and di-quark model of Jaffe and Wilczek.

In Skyrmon model^{10), 11)} penta-quark baryon is regarded as a single topological soliton. The soliton including Θ is assigned to pure $\overline{\mathbf{10}}$ representation of $SU(3)_f$. Mass of Θ is calculated to be 1530 MeV. The quark contents of the $\overline{\mathbf{10}}$ representation together with the average of the total number of s quark for this representation are given below;

$$\left\{ \begin{array}{ll} \Theta & \sim |[ud][ud]\overline{s}\rangle, & \langle N_s \rangle = 1 \\ N & \sim \{ |[ud][ud]\overline{\mathcal{N}}\rangle + |[ud][\mathcal{N}s]\overline{s}\rangle + |[\mathcal{N}s][ud]\overline{s}\rangle \} / \sqrt{3}, & \langle N_s \rangle = 4/3 \\ \Sigma & \sim \{ |[ud][\mathcal{N}s]\overline{\mathcal{N}}\rangle + |[\mathcal{N}s][ud]\overline{\mathcal{N}}\rangle + |[\mathcal{N}s][\mathcal{N}s]\overline{s}\rangle \} / \sqrt{3}, & \langle N_s \rangle = 5/3 \\ \Xi & \sim |[\mathcal{N}s][\mathcal{N}s]\overline{\mathcal{N}}\rangle, & \langle N_s \rangle = 2. \end{array} \right. \quad (2.14)$$

^{*)} For example, $\Delta_s \sim (2m_{u,d} + m_s) - (3m_{u,d}) \sim (2m_\Lambda + 6m_\Sigma + 12m_\Sigma^*)/20 - (4m_N + 16m_\Delta)/20 = 1300 - 1170 = 130 \text{ MeV}$.

Among them, Θ is the lightest, but the mass differences are only $(1/3)\Delta_s$. In the di-quark model,¹²⁾ a pair of quarks is a constituent unit of hadrons, so that the configuration of penta-quark baryon is (di-quark)-(di-quark)-(anti-quark). The resulting $\overline{10}$ and 8 representation are considered to be ideally mixed. The lightest state in this scheme is $[[ud][ud]\overline{N}]$, which is identified with Roper resonance(1440 MeV). The state $\Theta = [[ud][ud]\overline{s}]$ is heavier than this by Δ_s . Masses of Θ of the two models are about 1.5 GeV.

§3. Quark rearrangement diagram, duality and selection rules —physical roles of junction

That hadron structure and hadron reaction are strongly related is clearly seen through duality and is well manifested by quark rearrangement diagrams^{21), 22)} Except for diffraction scattering corresponding to Pomeron exchange, presence or absence of resonances in s -channel and Regge poles exchanged in t -channel are dual.

3.1. Quark rearrangement diagram and duality

1) $\overline{K}N$ and KN duality and penta-quark baryon

The non-Pomeron part of $\overline{K}N$ scattering is given only by H type diagram, while the non-Pomeron part of KN scattering is only by X type one. Quark rearrangement diagram of $\overline{K}N$ and KN is depicted in Fig. 2. Intermediate state of s -channel of the former is contributed from ordinary baryons with $S = -1$, while that of the latter with $S = +1$ is never from penta-quark baryons but simply the ones obtained by s - u crossing of Fig.2(a). Note that penta-quark baryon has the structure not only of five quarks but it has three junctions($J\bar{J}J$) in SJM and never appears in s -channel of KN .

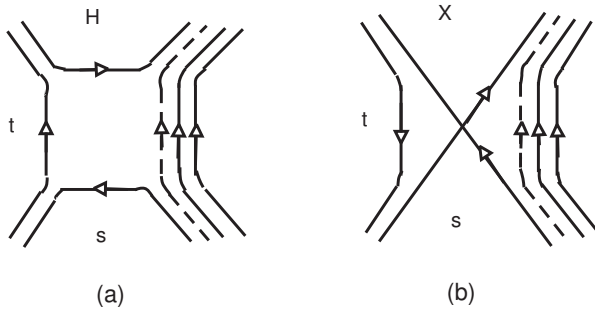


Fig. 2. (a) $\overline{K}N$ scattering diagram. Quarks are drawn by full lines. Junction is drawn by a dashed line. It is the H type diagram and is s - t -channel dual. (b) KN scattering diagram. It is the X type diagram and is u - t -channel dual. So it does not contain s -channel baryon resonance.

Total cross section difference $\Delta\sigma_T(KN)$ is related to the imaginary part of forward scattering amplitude T by optical theorem as below.

$$\begin{aligned}\Delta\sigma_T(KN) &\equiv \sigma_T(\overline{K}N) - \sigma_T(KN) = (1/s) (\text{Im}T(\overline{K}N) - \text{Im}T(KN)) \\ &= (1/s)(\text{Im}T(H) - \text{Im}T(X)) \sim (1/s)\text{Im}T(H).\end{aligned}\quad (3.1)$$

Here Pomeron contribution of diffraction scattering is cancelled between $\bar{K}N$ and KN scatterings. Thus all the imaginary parts $\text{Im}T$ in Eq. (3.1) should be understood to be the imaginary part of the respective “non-Pomeron” amplitude. That $\text{Im}T(KN) \equiv \text{Im}T(X) \sim 0$ strongly indicates absence of resonances in s channel on the one hand and corresponds to exchange degeneracy of meson Regge poles in t channel on the other. Then $\Delta\sigma_T(KN)$ is due to $\text{Im}T(\bar{K}N) \equiv \text{Im}T(H)$. This imaginary part owes to numbers of s channel resonances with $S = -1$ above $\bar{K}N$ threshold on the one hand, whose average coincides with t channel Regge poles known as finite energy sum rule^{23), 24)}. This $\Delta\sigma_T(KN)$ phenomenologically decreases as $s^{-1/2}$, where s is the square of the s channel energy. The factors $(1/s)$ in Eq.(3.1) are kinematical flux factor, and s dependence of $\text{Im}T(H) \sim s^{1/2}$ is given by the intercept of the meson Regge trajectories which, in turn, is interpreted by the number of the exchanged constituents, $N_q = 2$. Baryon resonances mediate only in u channel of KN scattering.

Idealized duality is elegantly expressed by Veneziano amplitude;²⁵⁾ s - t dual Veneziano amplitude $V(s, t)$ corresponds to H type diagram and u - t dual one $V(u, t)$ to X type one in Fig. 2, as summarized in Table III.

Table III. Duality of meson-baryon scattering

	H type	X type
s -channel resonances	yes	no
$\text{Im } T(t=0)$	yes	no
$\text{Re } T(t=0)$	no	yes
duality	s - t dual	u - t dual
Veneziano amplitude	$V(s, t)$	$V(u, t)$

2) $\bar{N}N$ and NN duality and unconventional mesons

Corresponding to Fig. 2(a) of $\bar{K}N$ scattering, there are three quark rearrangement diagrams of H type in $\text{SJM}^2)$ of $\bar{N}N$ scattering:

$$H_S(t : M, s : M_4^2), H_D(t : MM, s : M_2^2), H_T(t : MMM, s : S_0^2). \quad (3.2)$$

Note that the existence of junction leads to new diagrams $H_D(t : MM, s : M_2^2)$ and $H_T(t : MMM, s : S_0^2)$ in $\bar{N}N$ scattering. Here S, D, T mean respectively single, double and triple exchange of ordinary mesons in their t -channels. Note the contribution of unconventional mesons in their s -channels. There are also three s - t crossed ones;

$$\bar{H}_S(s : M, t : M_4^2), \bar{H}_D(s : MM, t : M_2^2), \bar{H}_T(s : MMM, t : S_0^2). \quad (3.3)$$

These six H type diagrams are shown in Fig. 3 (a) \sim (f).

We examine difference of total cross section $\Delta\sigma_T(NN)$ expressed as below:

$$\begin{aligned}
\Delta\sigma_T(NN) &\equiv \sigma_T(\bar{p}p) - \sigma_T(pp) = (1/s) \{ \text{Im}T(\bar{p}p) - \text{Im}T(pp) \} \\
&= (1/s) \{ \text{Im}(T(H_S) + T(H_D) + T(H_T) + T(\bar{H}_S) + T(\bar{H}_D) + T(\bar{H}_T)) \\
&\quad - \text{Im}(T(X_S) + T(X_D) + T(X_T) + T(\bar{X}_S) + T(\bar{X}_D) + T(\bar{X}_T)) \} \quad (3.4) \\
&\sim (1/s) \text{Im}(T(H_S) + T(\bar{H}_T)).
\end{aligned}$$

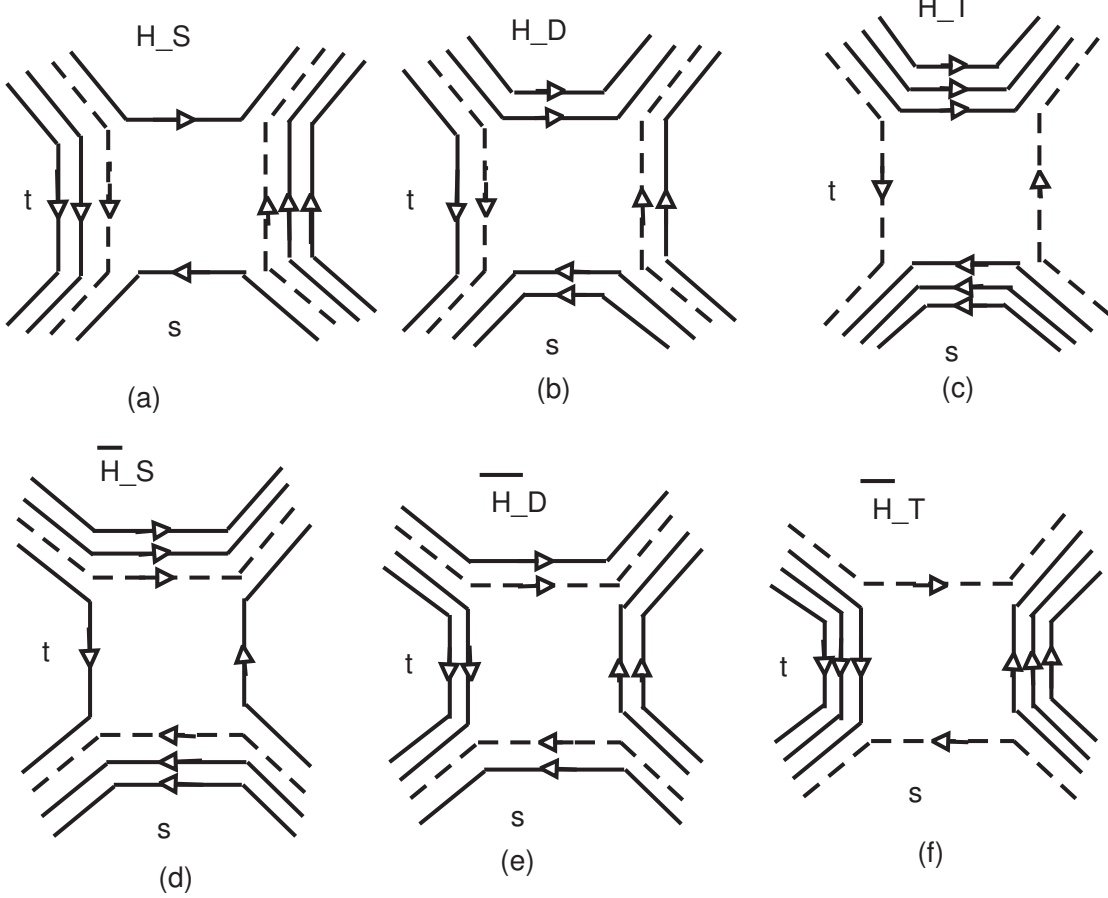


Fig. 3. H type diagrams in $\bar{N}N$ scattering. Quarks are drawn by full lines. Junctions are drawn by dashed lines. (a) $H_S(t : M, s : M_4^2)$, (b) $H_D(t : MM, s : M_2^2)$, (c) $H_T(t : MMM, s : S_0^2)$, and their s - t crossed diagrams: (d) $\bar{H}_S(s : M, t : M_4^2)$, (e) $\bar{H}_D(s : MM, t : M_2^2)$, (f) $\bar{H}_T(s : MMM, t : S_0^2)$

Similarly to Eq.(3.1), $\text{Im}T$ denotes the respective non-Pomeron amplitudes.

A few comments are in order. They are essentially the same as those on $\Delta\sigma_T(KN)$. Non-Pomeron part of $T(pp)$ is known to be almost real. It is given by six X -type diagrams, which are analogous to X type diagram in Fig.2(b) of KN scattering. They are respectively obtained from six H type diagrams in Fig. 3 by s - u crossing, whose s -channel states are never di-baryon resonances. Note that “di-baryon” in SJM is composed not only from six quarks but also from three junctions and one anti-junction, as shown in Fig.1(h). Di-baryon, thus, does not contribute

to X type diagrams in NN scattering. Two of six diagrams, $H_S(t : M, s : M_4^2)$ and $\overline{H}_T(s : MMM, t : S_0^2)$, are supposed to give the leading contribution with $s^{1/2}$ to $\text{Im } T(\overline{p}p)$, since the least number of constituents are exchanged in t -channel: (q, \overline{q}) -exchange in $H_S(t : M, s : M_4^2)$ and (J, \overline{J}) -exchange²⁾²⁶⁾ in $\overline{H}_T(s : MMM, t : S_0^2)$.

Although it is indirect evidence, duality gives important information about the mass of tetra-quark mesons. That $\Delta\sigma_T(NN)$ is phenomenologically large “above $\overline{p}p$ threshold” would surely indicate contributions of tetra-quark mesons M_4^2 in the s channel of $H_S(t : M, s : M_4^2)$ and imply that many of their masses are above but near to $2m_B$ or that δ is small negative or positive. But continuum states MMM of $\overline{H}_T(s : MMM, t : S_0^2)$ also give the leading contribution, which would mask the possible separated resonance peaks of M_4^2 .

3.2. Selection rules and penta-quark baryon

In order to secure a suppression of the decay of tetra-quark meson $M_4 \rightarrow MM$, Freund, Waltz and Rosner proposed a selection rule.¹³⁾ In SJM this FWR selection rule is interpreted as suppression of junction-anti-junction hair pin diagram^{14), 2)} similarly to OZI rule¹⁵⁾¹⁶⁾¹⁷⁾ to suppress quark-anti-quark hair pin diagram. We will call the process shown in Fig. 4(b) as “junction hair pin(FWR) rule” forbidden process.

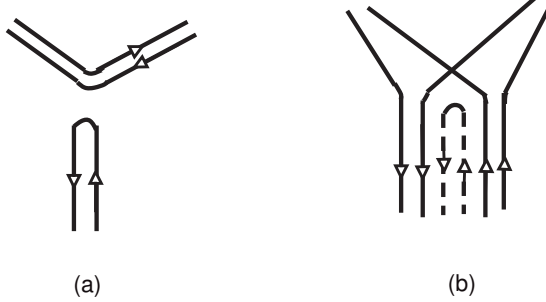


Fig. 4. (a) is OZI forbidden decay(e.g., $\phi \rightarrow \rho\pi$). It contains quark hair pin diagram. (b) is the process $M_4^2 \rightarrow MM$ and is “junction hair pin (FWR) rule” forbidden decay. It contains junction hair pin diagram. Junction is drawn by a dashed line. $B_5^3 \rightarrow MB$ occurs through a similar process.

The junction hair pin(FWR) rule is viewed from a different standpoint. When we apply $1/N_c$ expansion²⁷⁾²⁸⁾ to SJM, we obtain²⁹⁾ a suppression factor of $O(1/N_c)$ for OZI rule and that of $O(1/\sqrt{N_c})$ for junction hair pin(FWR) rule.

We conclude this subsection by saying that (1) K^+n scattering contains only X type amplitude so that no resonance contribution is expected in this case due to duality and (2) even if Θ exists, $J\overline{J}$ hair pin line has to be newly created in $Kn \rightarrow \Theta$, which is a forbidden process.

§4. Comments on parameters of SJM

4.1. Mass of junction

One simple way to estimate junction mass m_J is to use the mass of ordinary baryon and meson as follows,

$$m_B = 3m_q + m_J, \quad (4.1)$$

$$m_M = 2m_q. \quad (4.2)$$

If we estimate m_q from ρ and ϕ meson mass^{*)} as $m_M = 2m_q \sim (3m_\rho + m_\phi)/4 = 773$ MeV, $m_J \sim m_B - 3m_q \sim 1172 - 1160 = 12$ MeV, where m_B is estimated from the average of nucleon and Δ_{33} resonance^{**)}. The obtained value

$$m_J \sim O(10) \text{ MeV} \quad (4.3)$$

is quite small, $m_J \ll m_B, m_M$.

4.2. Naive estimate of length of string from uncertainty relation.

Every string in the ground state hadron in SJM is expected to have “minimal length” due to zero point oscillation. The minimal length and the mass are evaluated by uncertainty relation. Let x be the distance from a junction to a source quark with mass \tilde{m} and assume that the string connecting them is almost straight. The sum of kinetic energy of the source quark and linear potential energy to trap it is the constituent quark mass m_q , but we write it as E in this subsection for the sake of convenience.

$$E = \sqrt{p^2 + \tilde{m}^2} + \sigma \cdot x, \quad (4.4)$$

where p is the source quark momentum and σ is the string tension.

The string length L is estimated from x such that it minimizes E by taking $p = 1/x$ from the uncertainty relation.

(i) For light quarks $\mathcal{N} = (u, d)$ for which $\tilde{m}_{\mathcal{N}} \ll p$, $E_{\mathcal{N}}$ is approximated as

$$E_{\mathcal{N}} \sim p + (1/2)\tilde{m}_{\mathcal{N}}^2/p + \sigma \cdot x. \quad (4.5)$$

By putting $\tilde{m}_{\mathcal{N}} = 0$, it is minimized at $x = 1/\sqrt{\sigma}$, so

$$L_{\mathcal{N}} \sim 1/\sqrt{\sigma} \quad \text{and} \quad E_{\mathcal{N}} \sim 2\sqrt{\sigma}. \quad (4.6)$$

^{*)} Pseudoscalar meson should be discussed quite differently, since chiral symmetry and its spontaneous breaking plays essential role there.

^{**) $m_B \sim (4m_N + 16m_\Delta)/20 \sim 1172$ MeV.}

When we set the constituent quark mass $m_{\mathcal{N}} \equiv E_{\mathcal{N}}$ equal to $m_B/3=0.390$ GeV in the previous subsection by neglecting m_J , we obtain

$$\sqrt{\sigma} \sim 0.195 \text{ GeV}, \quad (4.7)$$

so that $L_{\mathcal{N}} \sim 5.1 \text{ GeV}^{-1}$.

(ii) For heavy quarks $Q = (c, b, t)$ whose masses $\tilde{m}_Q \gg p$, $E_Q \sim \tilde{m}_Q + (1/2)(p^2/\tilde{m}_Q) + \sigma \cdot x$, which is minimized at $x \sim 1/(\sigma\tilde{m}_Q)^{1/3}$, so

$$L_Q \sim 1/(\sigma\tilde{m}_Q)^{1/3} \quad \text{and} \quad E_Q \sim \tilde{m}_Q \{1 + (3/2)(\sigma/\tilde{m}_Q^2)^{2/3}\}. \quad (4.8)$$

Most of the constituent mass $m_Q \equiv E_Q$ comes from the source quark mass \tilde{m}_Q , and thus the mass difference among constituent quark masses comes almost from that of the source quarks.

It will be interesting to compare the above result with the one obtained by WKB approximation for $m_Q \equiv E_Q$ trapped in one dimensional linear potential. For linear potential

$$V(x) = \begin{cases} V_0 & ; \quad |x| > a, \\ (V_0/a)|x| & ; \quad |x| < a, \end{cases} \quad (4.9)$$

where $V_0 > 0$. WKB approximation gives

$$E_{\text{WKB}} \sim \tilde{m}_Q \{1 + (9\pi^2/128)^{1/3}(\sigma/\tilde{m}_Q^2)^{2/3}\}, \quad (4.10)$$

in the limit $V_0 \rightarrow \infty, a \rightarrow \infty$, with $V_0/a = \sigma$ kept finite. Note that both $E_Q - \tilde{m}_Q$ and $E_{\text{WKB}} - \tilde{m}_Q$ have the same functional form with different numerical factors: $(E_Q - \tilde{m}_Q) \sim 1.7(E_{\text{WKB}} - \tilde{m}_Q)$.

(iii) Intermediate of the above two cases is strange quark, whose source quark mass is $\tilde{m}_s = O(1) \times \sqrt{\sigma} \sim p$. Although we have to resort to a numerical calculation finally, let us use the Taylor expansion of $E_s = p + (1/2)\tilde{m}_s^2/p + \sigma \cdot x$ as Eq. (4.5), but the second term \tilde{m}_s^2/p is fixed in minimizing E_s by uncertainty relation.^{*)} Then $p \sim \sqrt{\sigma}$ and $E_s \sim 2\sqrt{\sigma} + (1/2)\tilde{m}_s^2/\sqrt{\sigma}$.

Here we get a somewhat surprising result as follows: The mass difference of constituent quarks from the above is $\Delta_s \equiv E_s - E_{\mathcal{N}} \sim (1/2)\tilde{m}_s^2/\sqrt{\sigma}$. The numerical value $\Delta_s=130\sim 150$ MeV is not obtained for $\tilde{m}_s \sim 130 \sim 150$ MeV. Namely, for $\sqrt{\sigma} = 0.195$ GeV, we have $\Delta_s \equiv E_s - E_{\mathcal{N}} \sim (1/2)\tilde{m}_s^2/\sqrt{\sigma} \sim 43$ MeV for $\tilde{m}_s = 130$ MeV and $\Delta_s \equiv E_s - E_{\mathcal{N}} \sim (1/2)\tilde{m}_s^2/\sqrt{\sigma} \sim 58$ MeV for $\tilde{m}_s = 150$ MeV. To reproduce the observed value of $\Delta_s \sim 130 \sim 150$ MeV, we have to take

$$\tilde{m}_s \sim 2\Delta_s. \quad (4.11)$$

Our numerical results are given in Table IV. This is obtained from the minimum of E of Eq. (4.4) with $p = 1/x$. Table IV includes the case of Eq. (4.11).

^{*)} For $\tilde{m}_s \sim 0.13 \text{ GeV} \sim 0.7 \times \sqrt{\sigma}$, we are able to confirm that \tilde{m}_s/p with $p \sim \sqrt{\sigma}$ is within convergence radius of Taylor expansion.

Table IV. String length and energy

quark	\bar{m}_q (GeV)	L_q (GeV ⁻¹)	E_q (GeV)	$\Delta_q = E_q - E_{\mathcal{N}}$ (GeV)
\mathcal{N}	0	5.13	0.390	0
s	0.130	4.73	0.428	0.04
s	0.270	4.18	0.520	0.13
c	1.35	2.66	1.50	1.11

For s quark $L_s \sim 0.8L_{\mathcal{N}}$. For heavy quarks L_Q is much shorter. For example, $L_c \sim 0.5L_{\mathcal{N}}$ for charm quark, though the $1/x$ behavior of the potential near $x \sim 0$ has to be taken into account of.

4.3. Comments on parameter δ

- (1) A direct way to estimate the parameter δ is to find an exotic meson M_4^2 ;

$$\delta = 2m_B - m(M_4^2).$$

We estimated it in 1977 as $\delta \sim 230 \sim 240$ MeV from the mass of M_4^2 candidate reported at that time^{18)*}). See, however, the next subsection newly added.

- (2) $\overline{B}B$ duality

An indirect way to estimate δ is to consider the $\overline{B}B$ duality. Above the threshold of $\overline{B}B$ -channel, the large difference $\Delta\sigma_T$ of Eq.(3·4) may imply that numbers of M_4^2 above the threshold contribute to $\text{Im}T(\overline{p}p)$, so that $\delta \sim$ small positive or negative.

4.4. Tetra-quark meson of Belle experiment, parameter m_{IJ} and δ in SJM

The Belle Collaboration has reported⁷⁾ a new resonance $Z^+(4430)$ in the invariant mass distribution of $\pi^\pm\psi'$. We consider this $Z^+(4430)$ is a good candidate for $M_4^2(ucJ; \bar{J}\bar{c}\bar{d})$ in SJM. By this assignment we can “directly” determine the parameter m_{IJ} , energy of inter-junction string, and thus δ .

The constituent quark mass m_q is set equal to E_q in Table IV as:

$$m_{\mathcal{N}} \equiv E_{\mathcal{N}}, \quad m_s \equiv E_s \quad \text{and} \quad m_c \equiv E_c. \quad (4.12)$$

If we adopt 4430 MeV as an input for the mass of $M_4^2(ucJ; \bar{J}\bar{c}\bar{d})$, we can set

$$2m_{\mathcal{N}} + 2m_c + m_{IJ} = 4430 \text{ MeV(input)}. \quad (4.13)$$

We obtain

$$m_{IJ} = 4430 - 780 - 3000 = 650 \text{ MeV}. \quad (4.14)$$

In the estimate, junction mass m_J and source quark masses of u, d quarks are neglected. We also find

$$\delta = 2m_{\mathcal{N}} - m_{IJ} = 780 - 650 = 130 \text{ MeV}. \quad (4.15)$$

*) See Table IV (p.143) in section IV of Ref. 30).

By these m_{IJ} and δ and values of Table IV, we give masses of unconventional hadrons. Especially, mass of penta-quark baryon $\Theta = B_5^3(udJ; \bar{s}\bar{J}; Jud)$ is

$$m(\Theta(B_5^3(udJ; \bar{s}\bar{J}; Jud))) = 4m_{\mathcal{N}} + m_s + 2m_{IJ} = 3380 \text{ MeV}, \quad (4.16)$$

which is much heavier than the value given by Skyrminion model¹⁰⁾¹¹⁾ or di-quark model.¹²⁾ Masses of various unconventional hadrons are listed below in MeV.

Unconventional meson M_4^2 :

$$\begin{aligned} Z^+ = M_4^2(ucJ; \bar{J}\bar{c}\bar{d}) &= 2m_{\mathcal{N}} + 2m_c + m_{IJ} = 4430(\text{input}) \\ M_4^2(\mathcal{N}\mathcal{N}J; \bar{J}\bar{\mathcal{N}}\bar{\mathcal{N}}) &= 4m_{\mathcal{N}} + m_{IJ} = 2210, \\ M_4^2(usJ; \bar{J}\bar{s}\bar{d}) &= 2m_{\mathcal{N}} + 2m_s + m_{IJ} = 2470. \end{aligned} \quad (4.17)$$

Unconventional meson M_2^2 :

$$\begin{aligned} M_2^2(\mathcal{N}J; \bar{J}\bar{\mathcal{N}}) &= 2m_{\mathcal{N}} + 2m_{IJ} = 2080 \\ M_2^2(cJ; \bar{J}\bar{c}) &= 2m_c + 2m_{IJ} = 4300. \end{aligned} \quad (4.18)$$

Unconventional meson S_0^2 (flavor singlet gluonium):

$$S_0^2(J; \bar{J}) = 3m_{IJ} = 1950. \quad (4.19)$$

Unconventional baryon B_5^3 :

$$\begin{aligned} B_5^3(\mathcal{N}\mathcal{N}J; \bar{\mathcal{N}}\bar{J}; J\mathcal{N}\mathcal{N}) &= 5m_{\mathcal{N}} + 2m_{IJ} = 3250, \\ \Theta = B_5^3(udJ; \bar{s}\bar{J}; Jud) &= 4m_{\mathcal{N}} + m_s + 2m_{IJ} = 3380. \end{aligned} \quad (4.20)$$

Unconventional baryon B_3^3 :

$$B_3^3(\mathcal{N}\mathcal{N}J; \bar{J}J\mathcal{N}) = 3m_{\mathcal{N}} + 3m_{IJ} = 3120. \quad (4.21)$$

Di-baryon D_6^4 :

$$D_6^4(\mathcal{N}\mathcal{N}J; \mathcal{N}\mathcal{N}J; \bar{J}; \mathcal{N}\mathcal{N}J) = 6m_{\mathcal{N}} + 3m_{IJ} = 4290. \quad (4.22)$$

In the Belle Collaboration, decay width of $Z^+(4430)$ is reported as “relatively narrow”:

$$\Gamma = 45 \text{ MeV}. \quad (4.23)$$

In SJM, decay $Z^+(4430) \rightarrow \pi\psi'$ occurs only through forbidden process by junction hair pin(FWR) rule(see Fig.4(b)).

§5. Summary

We consider the junction as not merely a mathematical symbol but a physical entity playing a role of connector of orientable strings, and discussed the nature of

unconventional hadrons with skeleton-like structure of junctions. We introduce cluster hypothesis into SJM. Based on this picture, mass of hadrons with multiclusters is given,

$$\begin{aligned} m &= m_q N_q + m_J N_J + m_{IJ} N_{IJ} \\ &= m_B \cdot N_J - \delta \cdot N_{IJ}. \end{aligned} \quad (5.1)$$

Parameters m_q , m_J , m_{IJ} are estimated:

- 1) Minimal length and energy m_q of string with quark at the end are estimated by uncertainty relation in section 4.2.
- 2) We estimate the mass of junction as

$$m_J \sim O(10) \text{ MeV}, \quad (5.2)$$

which is quite small: $m_J \ll m_B, m_M$. Among many papers based on AdS/CFT which cite some sketches of string junction structure of ordinary baryon, we are interested in Imamura's paper that evaluates m_J to be quite small^{31), 32)}

- 3) By identifying $Z^+(4430)$ with $M_4^2(ucJ; \bar{J}\bar{c}\bar{d})$, we estimate the inter-junction energy m_{IJ} and the energy δ to cut the inter-junction string in subsection 4.4,

$$m_{IJ} = 650 \text{ MeV} \quad \text{and} \quad \delta = 130 \text{ MeV}. \quad (5.3)$$

Investigation of inter-junction energy, m_{IJ} , based on AdS/CFT and lattice gauge theory will be awaited.

- 4) Taking into account the smallness of δ and m_J , the mass of hadrons in SJM is given approximately by $N_J \cdot m_B$. Together with parameters given in Table IV, masses of hadrons including unconventional ones are predicted as in 4.4. For example, $\Theta = B_5^3(udJ; \bar{s}\bar{J}; Jud) = 4m_N + m_s + 2m_{IJ} = 3380 \text{ MeV}$. This value is larger than those given by Skyrmion model and di-quark model.

Recently one of the authors(S. O.) was informed from Tamagaki³³⁾ that he is making study about such universal repulsion among three baryons that is necessary to stabilize neutron stars under the mixture of hyperons. This is the extension of his work about the universal repulsion between two baryons(R. Tamagaki, Bulletin of the Institute of for Chemical Research, Kyoto Univ. , Vol. 60, No.2(1982),190). By noting flavor independence of these two universal repulsions, he attributes them to flavor independent nature of the junction of SJM.

Note Added

In the previous version of the paper, there was a mistake in the description related to penta-quark baryon belonging to $\overline{10}_f$. We corrected the presentation related to this point.

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